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Contribution of Externally Bonded FRP to Shear Capacity of Flexural Members

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Abstract

Fiber reinforced polymer (FRP) materials are continuing to show great promise for use in strengthening reinforced concrete (RC) structures. These materials are an excellent option for use as external reinforcing because of their light weight, resistance to corrosion, and high strength. Externally bonded FRP sheets have been used to increase moment capacity of flexural members and to improve confinement in compression members.

Investigations into the use of externally bonded FRP sheets for use in shear strengthening have also been conducted and have shown this to be a viable strengthening method. The objective of this study is to review the current research on shear strengthening with FRP and propose design algorithms to compute the contribution of FRP to the shear capacity of RC flexural members. Methods for computing the shear capacity based on the stress level to cause tensile fracture of the FRP sheet (which may be less than ultimate due to stress concentrations) and based on delamination of the sheet from the concrete surface are presented. Areas which have the potential for further development are also discussed.

Keywords: Bonded Reinforcement, Carbon FRP, Concrete Structures, Design Algorithms, Delamination, Shear Strengthening

1. Introduction

The use of externally bonded fiber reinforced polymer (FRP) reinforcement to strengthen RC structures is becoming an increasingly popular retrofit technique. The light weight and formability of FRP reinforcement make these systems easy to install. And since the materials used in these systems are non-corrosive, non-magnetic, and generally resistant to chemicals, they are an excellent option for external reinforcement.

FRP is a composite material generally consisting of carbon, aramid, or glass fibers in a polymeric matrix (e.g. epoxy resin). Among many options, this reinforcement may be in the form of preformed laminates or flexible sheets. The laminates are stiff plates or shells that come pre-cured and are installed by bonding the plate to the concrete surface with epoxy. The sheets are either dry or pre-impregnated with resin (pre-preg) and cure after installation onto the concrete surface. This installation technique is known as wet lay-up.

Strengthening with externally bonded FRP sheets has been shown to be applicable to many types of RC structures. Currently, this method has been implemented to strengthen such structures as columns, beams, slabs, walls, chimneys, tunnels, and silos. The uses of external FRP reinforcement may be generally classified as flexural strengthening, improving the confinement and ductility of compression members, and shear strengthening.

Several studies have been conducted to explain the behavior of externally bonded FRP sheets used to increase the moment capacity of flexural members. One limit to increasing the moment capacity is that eventually the shear capacity of the member is exceeded. In these situations, it has been shown that externally bonded FRP sheets may be used to increase the shear capacity as well. However, few studies have specifically addressed shear strengthening and design algorithms for computing the shear contribution of FRP sheets are not yet clear.

One of the difficulties with defining the shear contribution of FRP sheets is the wide variety of possible FRP shear reinforcement configurations. Figure 1 shows several of these configurations. There are options in deciding which surfaces will be bonded (Figure 1-a), whether to use continuous reinforcement or a series of strips (Figure 1-b), and whether mechanical anchorage is necessary (Figure 1-e). In addition, FRP is an anisotropic material characterized by high strength in the direction of the fiber orientation. The fibers may, therefore, be oriented in directions to best reinforce shear cracks (Figure 1-c) or it may be beneficial to create pseudo-isotropy by orienting the fibers in two perpendicular directions (Figure 1-d).

The aim of this study is to review the current research on shear strengthening with bonded FRP reinforcement and to present simple design algorithms for computing the contribution of FRP to shear strength of RC members. The research focuses on carbon FRP (CFRP) and seeks to address several of the CFRP shear reinforcement configurations. The current procedures do not address every aspect of FRP shear reinforcement, however research into comprehensive design procedures is continuing at the University of Missouri – Rolla (UMR).

2. Shear Strength of RC Beams Strengthened with FRP Reinforcement

The nominal shear strength of a RC beam may be computed by the basic design equation presented in ACI 318-95 and given below as Equation (1).

$$V_n = V_c + V_s \tag{1}$$

In this equation the nominal shear strength is the sum of the shear strength of the concrete (which for a cracked section is attributable to aggregate interlock, dowel action of the longitudinal reinforcement, and the diagonal tensile strength of the uncracked portion of concrete) and the strength of the steel shear reinforcement. In the case of beams strengthened with externally bonded FRP sheets, the nominal shear strength may be computed by the addition of a third term to account for the contribution of the FRP sheet to the shear strength. This is expressed in Equation (2).

$$V_n = V_c + V_s + V_f \tag{2}$$

The design shear strength is obtained by multiplying the nominal shear strength by a strength reduction factor, ϕ. It is suggested that the reduction factor of $\phi = 0.85$ given in ACI 318-95 be maintained for the concrete and steel terms. However, the reduction factor for CFRP reinforcement will require an adjustment as discussed later.

3. The Contribution of FRP Reinforcement to Shear Capacity (V_f)

In order to compute the nominal shear strength as given in Equation (2), it is necessary to quantify the contribution of CFRP reinforcement to the shear capacity (V_f). The contribution of CFRP depends on several parameters including the stiffness of the CFRP sheet, the quality of the epoxy resin, the compressive strength of the concrete, the number of layers of CFRP sheet, the wrapping scheme, and the fiber orientation angle. It has been difficult to establish one formula to compute V_f because the parameters are numerous and there is a lack of adequate experimental results. This study presents two equations that may be used to obtain V_f and suggests taking the lower of the two results as the shear strength contribution of the CFRP reinforcement. These two equations represent two possible failure modes.

The first of the two failure modes is CFRP rupture. Based on the experimental results and concepts presented by Triantafillou (Oct 1997), CFRP rupture occurs at an average stress level below the ultimate strength of CFRP due to stress concentrations. This average stress level, the effective stress, may be determined from experimental results comparing the stiffness of the CFRP sheet used in various strengthening efforts. Due to a lack of experimental results, this method does not, however, address the effect of concrete strength or bonded surface configuration on the shear capacity.

The other investigated mode of failure is delamination of the FRP from the concrete surface. The analytical approach used to model this mode of failure is based on the bond mechanism of the CFRP sheet to the concrete surface which was addressed by Maeda et al. (Oct 1997). The research conducted by Maeda is modified for use in computing the ultimate shear capacity of the CFRP sheet. This approach does include the effect of sheet stiffness, concrete strength, and bonded surface configuration.

3.1 Design Approach Based on the Effective FRP Stress

The design approach based on fracture of the CFRP sheet is quite similar to the approach used to compute the contribution of steel shear reinforcement. The stress in the sheet at ultimate must be calculated in the vertical direction and multiplied by the area of sheet that crosses a potential shear crack. However, instead of the ultimate condition being governed by a yield point, as with steel, the rupture point of the CFRP sheet must be considered. Triantafillou (Oct 1997) noted that CFRP sheets used for shear strengthening rupture at stress levels below their ultimate strength due to stress concentrations in the sheet. If the level of strain at rupture is considered as the effective strain, ε_{fe}, the contribution of externally bonded FRP sheets to the shear capacity of an RC beam may be computed from Equation (3).

$$V_f = \rho_f\, E_f\, \varepsilon_{fe}\, b_w\, 0.9d\,(1 + \cot\beta)\sin\beta \qquad (3)$$

This equation, as presented by Triantafillou, is in the Eurocode format. The shear reinforcement ratio, ρ_f, is the FRP shear reinforcement ratio as defined in the appendix, and the angle β is the angle between the orientation of the principal fibers in the sheet and the longitudinal axis of the beam. This equation may be rewritten in ACI code format as Equation (4).

$$V_f = \frac{A_f f_{fe}(\sin\beta + \cos\beta)d_f}{s_f} \qquad (4)$$

Here the effective strain times the modulus of elasticity is replaced with the effective stress. The area of CFRP shear reinforcement is the total thickness of the sheet (usually $2t_f$ for sheets on both sides of the beam) times the width of the CFRP strip. Note if continuous sheets are used (see Figure 1-b(i)) the width of the strip, w_f, and the spacing of the strips, s_f, should be equal.

3.1.1 Determination of the Effective Strain

In order to apply Equation (3) or (4), it is necessary to quantify the effective strain. Triantafillou observed the effective strain to be a function of the axial rigidity of the FRP sheet ($\rho_f E_f$). The effective strain was, therefore, determined by finding V_f experimentally for several rigidities of FRP sheet. Based on the experimental results, the effective strain was back calculated and plotted versus the axial rigidity. A relationship between effective strain and axial rigidity was found and is given in Equation (5-a) and (5-b).

$$\varepsilon_{fe} = 0.0119 - 0.0205\,(\rho_f E_f) + 0.0104\,(\rho_f E_f)^2 \quad \text{for } 0 \le \rho_f E_f \le 1 \text{ GPa} \qquad (5\text{-a})$$

$$\varepsilon_{fe} = 0.00245 - 0.00065\,(\rho_f E_f) \quad \text{for } \rho_f E_f > 1 \text{ GPa} \qquad (5\text{-b})$$

3.1.2. Modifications to the Effective Strain Model

Using the same approach as discussed above, but including additional experimental data that has become available (all available data is summarized in Table 1), the authors suggest a small modification to the model presented by Triantafillou. The model is further modified based on the observation that $\rho_f E_f$ does not exceed 1.1 GPa in all the data. Figure 2 shows the comparison between Equations (5-a) and (5-b) and the experimental data.

As originally suggested, the effective strain is determined by equating the experimentally determined shear strength to Equation (3) and back calculating ε_{fe}. To eliminate the effects of various types of FRP sheet, the ratio of effective strain to ultimate strain, $R = \varepsilon_{fe} / \varepsilon_{fu}$, is plotted versus axial rigidity. This plot is shown in Figure 3. A polynomial is used as a best fit to the data in the case of $\rho_f E_f < 1.1$ GPa. This polynomial is given in Equation (6).

$$R = 0.5622\,(\rho_f E_f)^2 - 1.2188\,(\rho_f E_f) + 0.778 \le 0.50 \qquad (6)$$

The upper limit on R of 0.50 has the effect of limiting the strain in the FRP sheet to an order of 400 $\mu\varepsilon$ to 500 $\mu\varepsilon$. This limit is suggested to maintain the shear integrity of the concrete. At higher levels of strain the shear crack widths would be such that aggregate interlock would be lost and the shear capacity of the concrete dramatically reduced. Note that this limit applies only to low-modulus CFRP sheets which have an ultimate strain on the order of 1.5%. Although an upper limit of 0.5 on a CFRP sheet with $\varepsilon_{fu} = 1.5\%$ would give an effective strain of 750 $\mu\varepsilon$, it should be noted that the additional application of a design resistance factor (as discussed in section 4.1) will further limit the strain to the order of 400 $\mu\varepsilon$ to 500 $\mu\varepsilon$ as previously discussed.

The ratio of effective strain to ultimate strain, R, may be used as a reduction factor on the ultimate strain. Thus, the effective strain for use in Equation (3) may be computed from Equation (7).

$$\varepsilon_{fe} = R\,\varepsilon_{fu} \qquad (7)$$

Since CFRP is linearly elastic until failure, the effective stress may be computed similarly by Equation (8) for use in Equation (4).

$$f_{fe} = R\, f_{fu} \tag{8}$$

Thus the computational procedure for design would involve finding the reduction factor, R, from Equation (6); finding the effective stress, f_{fe}, by Equation (8); and determining the shear contribution of the CFRP, V_f, from Equation (4).

3.1.3. Limitations of the Effective Stress Method

The experimental data used in Figure 3 includes two different types of fiber material (CFRP and AFRP), three bonded surface configurations (completely wrapped, U-jacket, and bonded on two sides), continuous reinforcement as well as strips, 45° and 90° fiber orientations, and single as well as double plies. Although all the data follow nearly the same trend (and only a single curve fit was proposed), a modification for each group may be required in the future when more experimental data becomes available. For example, there are only four available experiments with AFRP. This is not sufficient to warrant the use of this equation for AFRP reinforcement; the authors suggest this approach for CFRP only.

The fiber orientation angle in the experimental data is limited to 45° and 90°, whereas Equations (3) and (4) suggest application to all angles including 0°. More experimental data is required to validate the use of these equations for other fiber orientations.

Another observation is that the proposed equations do not take into consideration either the strength of the concrete or the bonded surface configuration. However, the bond between the FRP and concrete depends on, among other things, the concrete strength and the bonded area. Therefore, these equations are only applicable when failure is governed by FRP sheet rupture not by FRP sheet delamination.

3.2. Design Approach Based on the Bond Mechanism

Once shear forces develop inclined cracks in the concrete, high tensile stresses are developed in the portions of CFRP sheet that bridge these cracks. The tensile stresses in vertically oriented CFRP sheets are a result of the vertical separation of rigid bodies of concrete on either side of the crack. These tensile stresses must be transferred to the concrete on each side of the crack by interfacial bond stresses. If this interfacial bond is compromised before rupture of the CFRP sheet, a delamination failure occurs. FRP delamination becomes a crucial consideration, especially if the sheet is not wrapped around the beam entirely. In order to address the delamination failure mode, another approach based on the bond characteristics of CFRP sheets with concrete is presented. The approach will, necessarily, consider the effects of concrete strength and bonded surface configuration.

Maeda et al. (Oct 1997) studied the bond mechanism of CFRP sheets with concrete by performing simple tension tests. The use of this type of test to characterize the bond mechanism for shear strengthening is reasonable considering the mechanism of force transfer in a shear strengthening configuration. The study described presents the concept of average bond strength and effective bond length based on experimental results and suggests empirical equations to predict bond behavior.

Based on this study, the authors present a set of equations that apply the concepts of effective bond length and average bond stress to shear strengthening. The empirical equations used to predict the average bond strength and effective bond length may be used to quantify the ultimate capacity of CFRP sheets at delamination.

3.2.1. Effective Bond Length and Ultimate Load Capacity

The experiments conducted by Maeda et al. involved tensioning CFRP strips bonded to a concrete surface (see Figure 4). Experiments were run for various CFRP sheet stiffnesses and bonded lengths. According to observations by Maeda et al., for bonded lengths over 100 mm the ultimate tensile force that the CFRP strip carries is not dependent on its bonded length. The reason for this is that at an early stage of loading, load is sustained by bond in the vicinity of the loading point. If delamination occurs in this vicinity by concrete fracture, the area of active bonding is shifted to a new area. This action is repeated until delamination propagates completely through the length of the CFRP. Therefore, bond stresses are only transferred in the active bonding area. The length of CFRP that includes the active bonded area is termed the effective bond length, L_e.

Based on the experimental data acquired by Maeda et al., an exponential equation was proposed to predict the effective bond length. This equation is given below as Equation (9) and is a function of the thickness of the FRP sheet and the elastic modulus of the FRP. As the stiffness of the sheet increases, the effective bond length decreases.

$$L_e = e^{6.134 - 0.58 \ell n (t_f E_f)} \tag{9}$$

Further experimental data indicates that the bond stress at failure is a linear function of the stiffness. This relationship for the average bond strength, τ_{bu}, may be computed from Equation (10) where k is an experimental constant equal to 110.2×10^{-6} 1/mm.

$$\tau_{bu} = k\, E_f\, t_f \tag{10}$$

Finally, considering an active bonded area equal to the effective bond length times the width of the bonded sheet, the ultimate load capacity of the CFRP sheet, P_{max}, may be computed from Equation (11).

$$P_{max} = L_e\, w_f\, \tau_{bu} \tag{11}$$

The ultimate load that the CFRP sheet can carry when failure is governed by delamination is mainly a function of the stiffness of the CFRP sheet. This relationship is shown graphically in Figure 5.

3.2.2. Effect of Concrete Strength

In addition to the stiffness of the CFRP sheet, the bond strength depends on the concrete compressive strength. The concrete used in the experiments by Maeda et al was consistently 42 MPa. For concrete of other strengths, a modification is required.

According to the conclusions of Horiguchi et al. (Oct 1997), the bond strength between the FRP sheet and the concrete surface is a function of $(f'_c)^{2/3}$. The modification to Equation (10) may, therefore, be accomplished by multiplying by $(f'_c/42)^{2/3}$. This modification is reflected in Equation (12).

$$\tau_{bu} = k\, (f'_c/42)^{2/3}\, E_f\, t_f \tag{12}$$

3.2.3. Effect of Bonded Surface Configuration

For the case of shear strengthening, once a shear crack develops, only that portion of FRP extending past the crack by the effective bonded length will be capable of carrying shear. It is, therefore, suggested to replace the width of the FRP sheet, w_f, with an effective width, w_{fe}, in Equation (11). The effective width depends on the shear crack angle (assumed to be 45°) and the bonded surface configuration as shown in Figure 6. The value of w_{fe} may be computed from Equation (13).

$w_{fe} = d_f$ If the sheet is wrapped around the beam entirely (13-a)

$w_{fe} = d_f - L_e$ If the sheet is in the form of a U-jacket (13-b)

$w_{fe} = d_f - 2\,L_e$ If the sheet is bonded to only the sides of the beam (13-c)

In the case of CFRP strips, w_{fe} is equal to the sum of the width of all strips within the effective width defined by Equation (13). This may be accomplished by multiplying w_{fe} by w_f/s_f.

3.2.4. Proposed Bond-based Design Approach

The design approach based on the bond mechanism involves the calculation of the average bond stress from Equation (12) and the effective bond width from Equation (13). The contribution of the CFRP sheet to the shear capacity may then be calculated from Equation (14).

$$V_f = \frac{2L_e w_f \tau_{bu} w_{fe}}{s_f} \qquad (14)$$

3.2.5. Validity of the Proposed Bond-based Design Approach

In order to examine the validity of the proposed procedure, test results from other researchers are used (Table 1, tests 1 to 15). A comparison of the proposed design approach with the experimental results is given in Figure 7. The design approach was in good agreement with the test results and tends to give conservative results.

4. Design Considerations

The equations presented in the preceding section allow the computation of the nominal shear contribution of CFRP sheets to RC beams. There are further considerations that must be made when designing these systems; this section seeks to address those items.

4.1. Strength Reduction Factor

Due partly to the novelty of this repair technique, it is advisable to apply more stringent reduction factor to the FRP shear reinforcement than to the concrete shear strength and steel shear reinforcement. Triantafillou recommends a partial reduction factor of $1/\gamma = 1/1.15$ applied to Equation (3) in the Eurocode format. Based on the available experimental data, the authors suggest a reduction factor of $\phi = 0.70$ (ACI format) applied to V_f in Equations (4) and (14). A comparison of the experimental data with the shear contribution of FRP computed using this value is given in Figure 8. The figure shows this recommendation to be conservative for nearly all cases.

4.2. Pseudo-isotropic Reinforcement

The design equations presented do not address the use of pseudo-isotropic CFRP reinforcement where the principal fibers are oriented in two perpendicular directions. Although the effect of this reinforcement is not quantifiable at this time, its use is highly recommended. When shear cracks form, it is typically assumed that the displacement is in the vertical direction and the vertical component of the resistive force supplied by reinforcement is effective. However, in reality the displacement has a horizontal component as well resulting from rigid body rotation about the shear crack tip. If only vertical plies of FRP are used ($\beta = 90°$), there is nothing to resist this horizontal strain component. (In the case of steel stirrups, this component is resisted by dowel action of the stirrup.) It is, therefore, recommended to use an additional horizontal ply ($\beta = 0°$) to resist this movement.

Research into quantifying this effect is currently being conducted at UMR.

4.3. Design Protocol

The procedure for design as presented in this paper is summarized by the flowchart given in Figure 9.

4.4. Solved Example

For the beam cross section with dimensions shown in Figure 10, the contribution of one ply of CFRP sheet reinforcement to the ultimate shear capacity is desired. The FRP is attached to the beam in its shear span by epoxy resin and in a U-jacket bonded surface configuration.

Data: The concrete strength is 27MPa, the modulus of elasticity of the CFRP sheet is 227 GPa, the ultimate axial stress of the CFRP sheet is 3400 MPa, and the sheet thickness is 0.165 mm/ply. The angle of fiber orientation is 90°. The steel stirrups are 10M bars (100 mm^2) with a yield strength of 300 MPa. The spacing of the stirrups is 200 mm.

Solution 1 (based on the effective FRP strain)

$\rho_f = 2\, t_f / b_w = 2\,(0.165)/150 = 0.0022$

$\rho_f\ E_f = 0.0022 \times 227 = 0.5$ GPa

$A_f = 2 \times 0.165 \times 1000 = 330\ mm^2$ (Based on a unit width of 1 m)

Using equation (6)

$R = 0.5622\ (0.5)^2 - 1.218\ (0.5) + 0.778 = 0.31 < 0.50$ (Satisfied)

$f_{fe} = 0.31 \times 3400 = 1054$ MPa = 1.054 GPa

Using equation (4)

$V_f = (330 \times 1.054 \times 1 \times 450)/1000 = 156$ kN

Solution 2 (based on bond mechanism)

Use equation (9) to calculate L_e

$L_e = 2.7183^{(6.134 - 0.58 \ln(0.165 \times 227))} = 56.4$ mm

Use equation (12) to calculate τ_{bu}

$\tau_{bu} = 227 \times 0.165 \times 110.2 \times 10^{-6} \times (27/42)^{2/3} = 0.00307$ GPa

In this case, $w_{fe} = d_f - L_e = 450 - 56.4 = 393.6$ mm

Apply equation (14) to calculate V_f

$V_f = 2 \times 56.4 \times 393.6 \times 0.00307 = 136$ kN

The lowest of the two results is taken as the controlling value. Therefore, the shear contribution of the CFRP sheet in this example is:

V_f=136 kN controlled by CFRP sheet delamination

The total shear capacity of the beam may then be computed.

$$V_c = \frac{\sqrt{f'_c}\, b_w d}{6} \text{N} = \frac{\sqrt{27}(150)(550)}{6} = 71447 \text{ N} = 71.5 \text{ kN}$$

$$V_s = \frac{A_v f_y d}{s} = \frac{2(100)(300)(550)}{200} = 165000 \text{ N} = 165 \text{ kN}$$

$V_n = V_c + V_s + V_f = 71.5$ kN + 165 kN + 136 kN = 372.5 kN

$\phi V_n = 0.85(V_c + V_s) + 0.70V_f = 0.85(71.5 + 165) + 0.70(136) = \underline{296.2 \text{ kN}}$

Thus, a 47% increase in shear capacity is achieved with the addition of CFRP reinforcement.

5. Conclusions

Based on the experimental and analytical results collected from other researchers, two design approaches are presented to calculate the contribution of externally CFRP sheet (V_f) to the shear capacity of an RC beam.

The first approach is based on the effective FRP stress, which is presented in this study as a function of its stiffness and ultimate strain. This design approach is valid for CFRP continuous sheets or strips and for any fiber orientation angle. It is only suitable if the failure is controlled by FRP sheet rupture.

The second approach is based on the bond mechanism between the CFRP sheet and the concrete. The effective width of FRP sheet at delamination is addressed. This design approach is valid only for CFRP continuous sheets or strips and is suitable if the failure is controlled by CFRP sheet de-lamination.

These two design methods are easy to apply, and a solved example is presented. The authors suggest applying the two design methods and taking the lowest resulting value of V_f.

To prove the validity of the design approach, further experimental and analytical work is needed. The recommended further studies must address the following:

1. Most of the current experimental available work is for the case of FRP wrapped entirely around the beam. Experimental studies are needed for case of the more practical U- jacket configuration.
2. Experimental work is needed to show the effect of FRP orientation on the capacity.
3. The effect of adding one ply of FRP sheet in 0° direction over a ply in the 90^0 direction needs to be investigated.
4. Experimental and analytical work is needed to address the issue of bond between FRP and concrete. The effect of concrete strength and FRP orientation on the effective bond length and average bond strength needs more study.
5. Work on the effect of anchoring shear reinforcement with mechanical type anchors is needed.
6. The effect of the presence and amount of steel shear reinforcement on the total shear capacity needs to be investigated.

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Appendix: Notation

A_f = area of CFRP shear reinforcement = 2 t_f w_f

b_w = width of the beam cross section

d = depth from the top of the section to the tension steel reinforcement centroid

d_f = effective depth of the CFRP shear reinforcement (usually equal to d for rectangular sections and d-t_s for T-sections)

E_f = elastic modulus of FRP (GPa)

f'_c = nominal concrete compressive strength (MPa)

f_{fe} = effective tensile stress in the FRP sheet in the direction of the principal fibers

f_{fu} = ultimate tensile strength of the FRP sheet in the direction of the principal fibers

k = experimental constant describing the gradient of the effective bond length = $110.2x10^{-6}$ 1/mm

L_e = effective bond length (mm)

P_{max} = ultimate load carried by CFRP sheet

R = ratio of effective stress or strain in the FRP sheet to its ultimate strength or elongation

s_f = spacing of FRP strips

t_f = thickness of the FRP sheet on one side of the beam (mm)

t_s = slab thickness

V_c = nominal shear strength provided by concrete

V_f = nominal shear strength provided by FRP shear reinforcement

V_n = nominal shear strength

V_s = nominal shear strength provided by steel shear reinforcement

w_f = width of FRP strip

w_{fe} = effective width of FRP sheet (mm)

β = angle between the principal fiber orientation and the longitudinal axis of the beam

ε_{fe} = the effective FRP strain

ε_{fu} = ultimate tensile elongation of the fiber material in the FRP composite

ϕ = strength reduction factor

ρ_f = FRP shear reinforcement ratio = $(2t_f / b_w) (w_f / s_f)$

τ_{bu} = average bond strength (GPa)

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Figures

Tables

Khalifa, A., W.J. Gold, A. Nanni, and M.I. Abdel Aziz, "Contribution of Externally Bonded FRP to Shear Capacity of Flexural Members" ASCE-Journal of Composites for Construction, Vol. 2, No.4, Nov. 1998, pp. 195- 203.

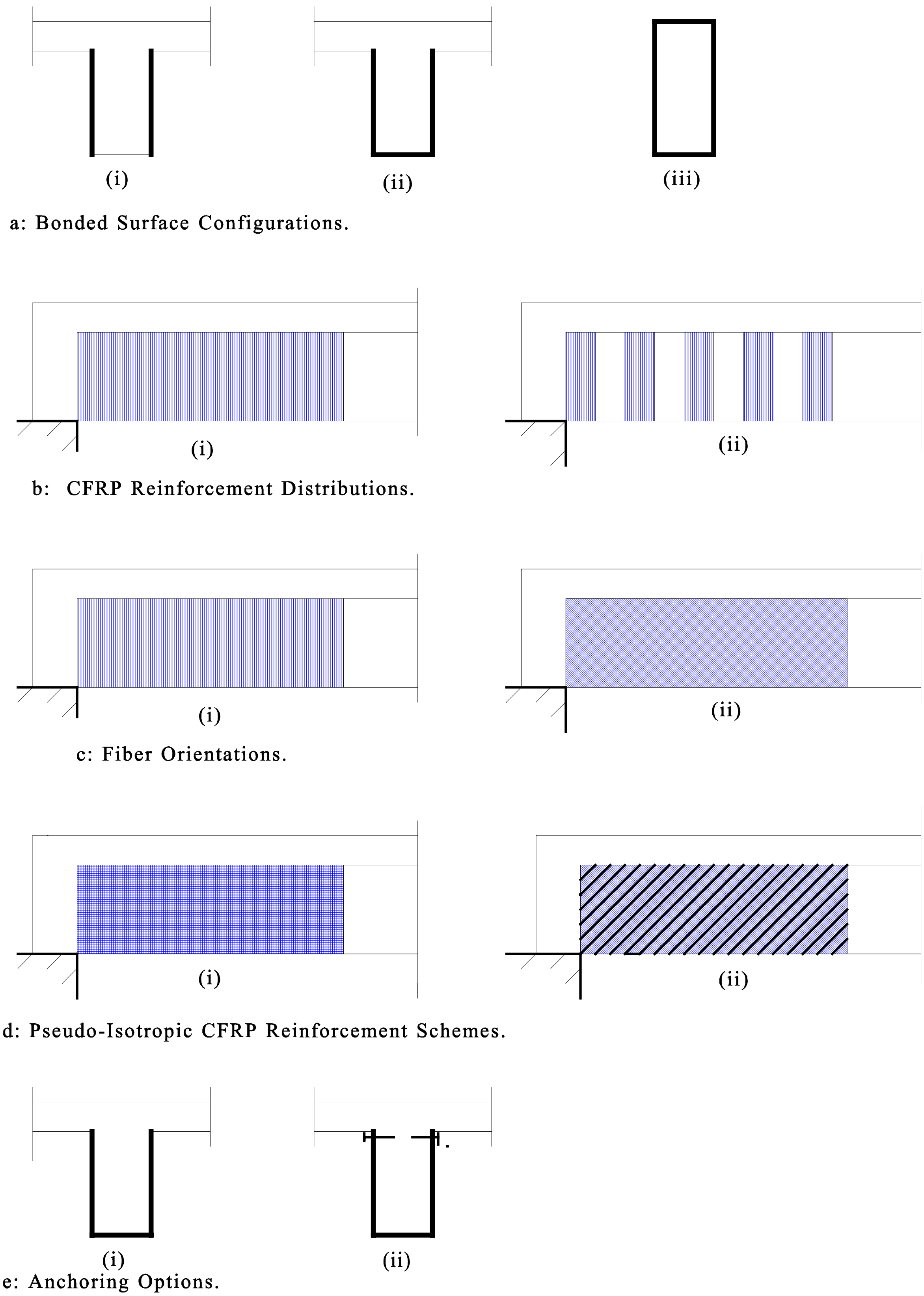

Fig.1.FRP Shear Reinforcement Configurations

Khalifa, A., W.J. Gold, A. Nanni, and M.I. Abdel Aziz, "Contribution of Externally Bonded FRP to Shear Capacity of Flexural Members" ASCE-Journal of Composites for Construction, Vol. 2, No.4, Nov. 1998, pp. 195- 203.

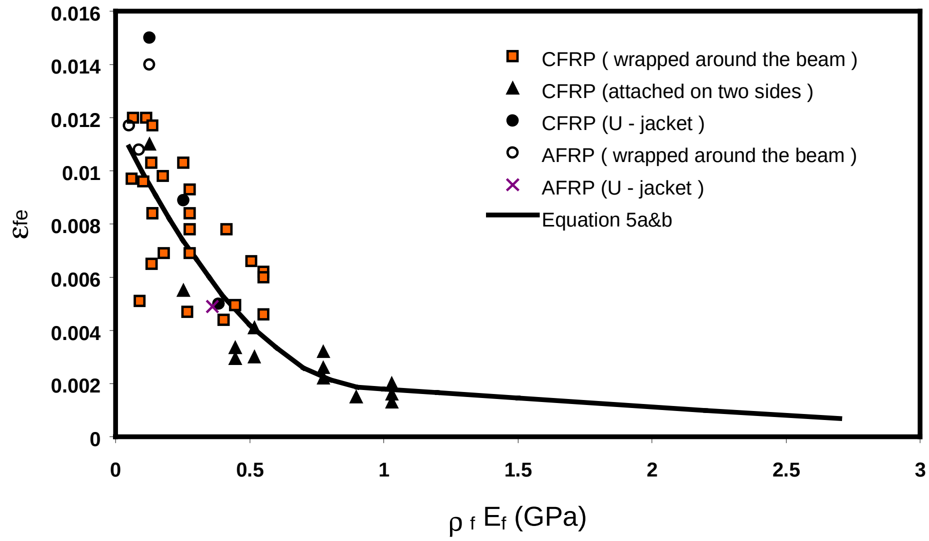

Fig.2. Comparison Between Equation 5a&b and Experimental Results (Original Triantafillou)

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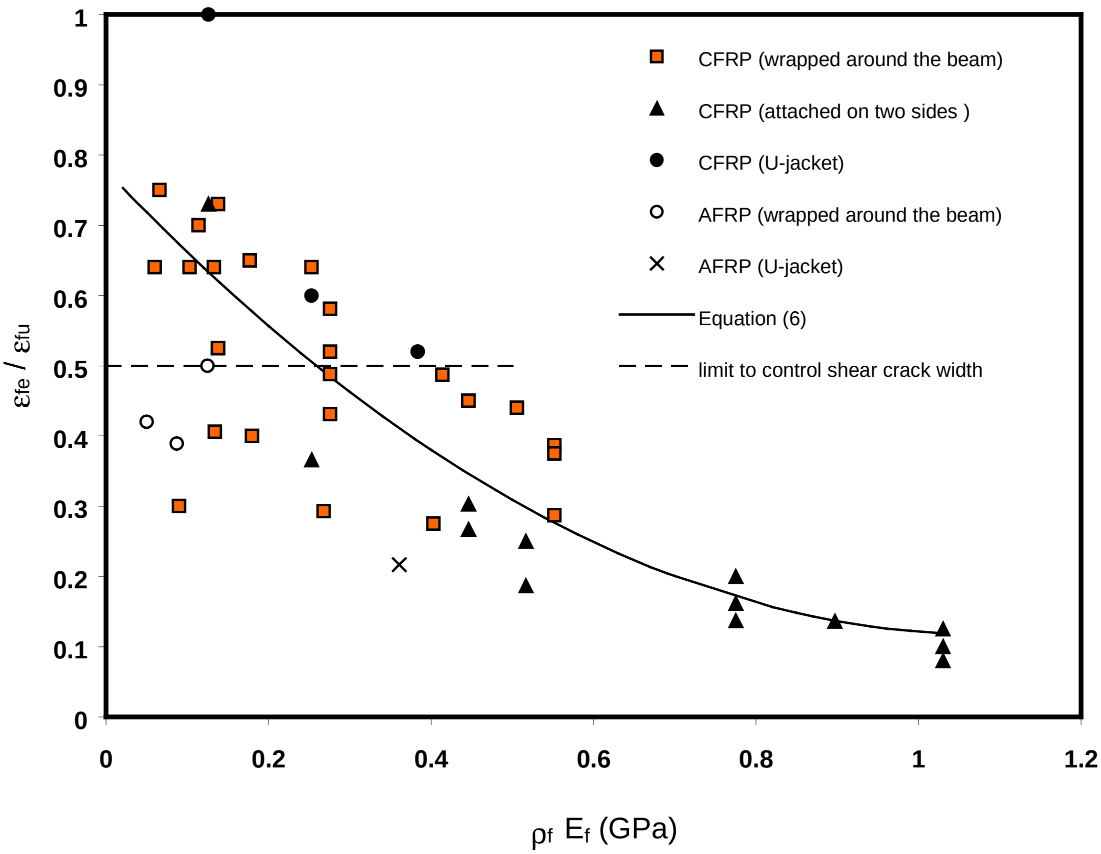

Fig.3. Ratio of ε_{fe} / ε_{fu} in terms of ρ_f E_f

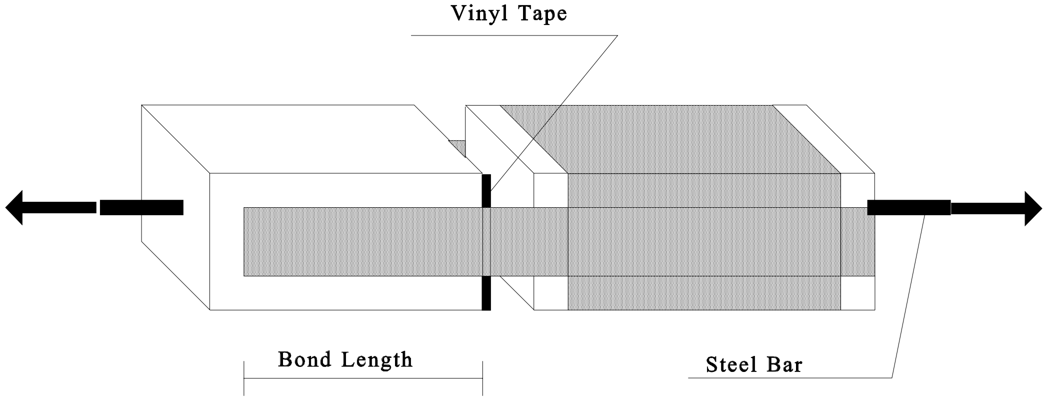

Fig. 4 . Sketch of Bond Test Configuration

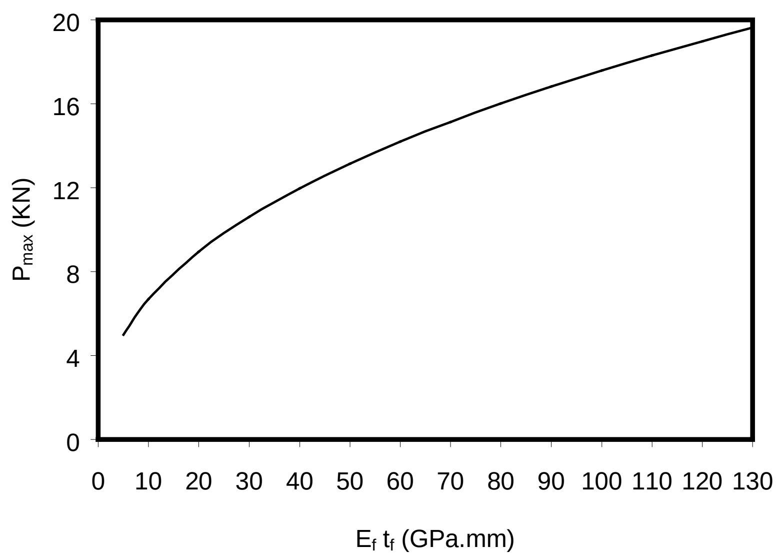

Fig.5.Plot of Sheet Stiffness Versus Ultimate Load Capacity at Delamination (Based on Maeda Equations and CFRP Sheet Width Equal 50 mm)

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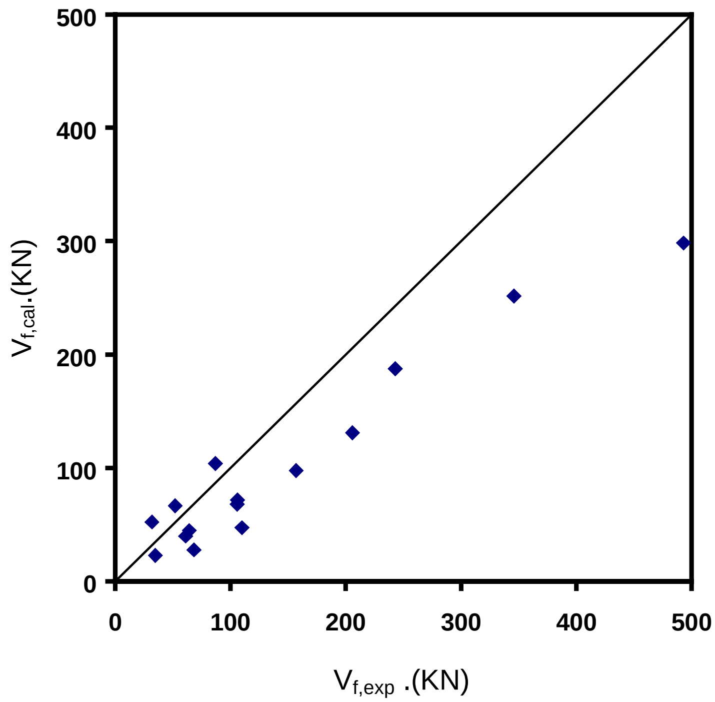

Fig. 7 . Comparison of Calculated and Experimental Results

Khalifa, A., W.J. Gold, A. Nanni, and M.I. Abdel Aziz, "Contribution of Externally Bonded FRP to Shear Capacity of Flexural Members" ASCE-Journal of Composites for Construction, Vol. 2, No.4, Nov. 1998, pp. 195- 203.

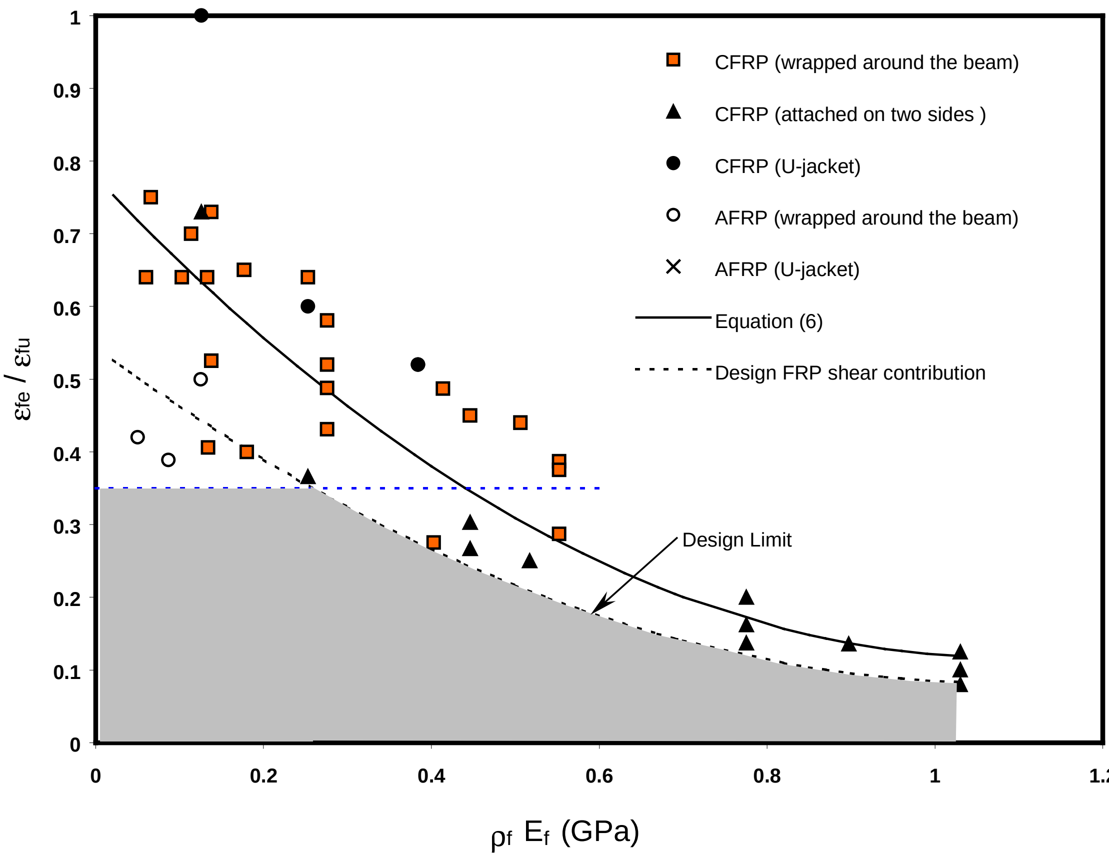

Fig.8. Effect of Strength Reduction Factor

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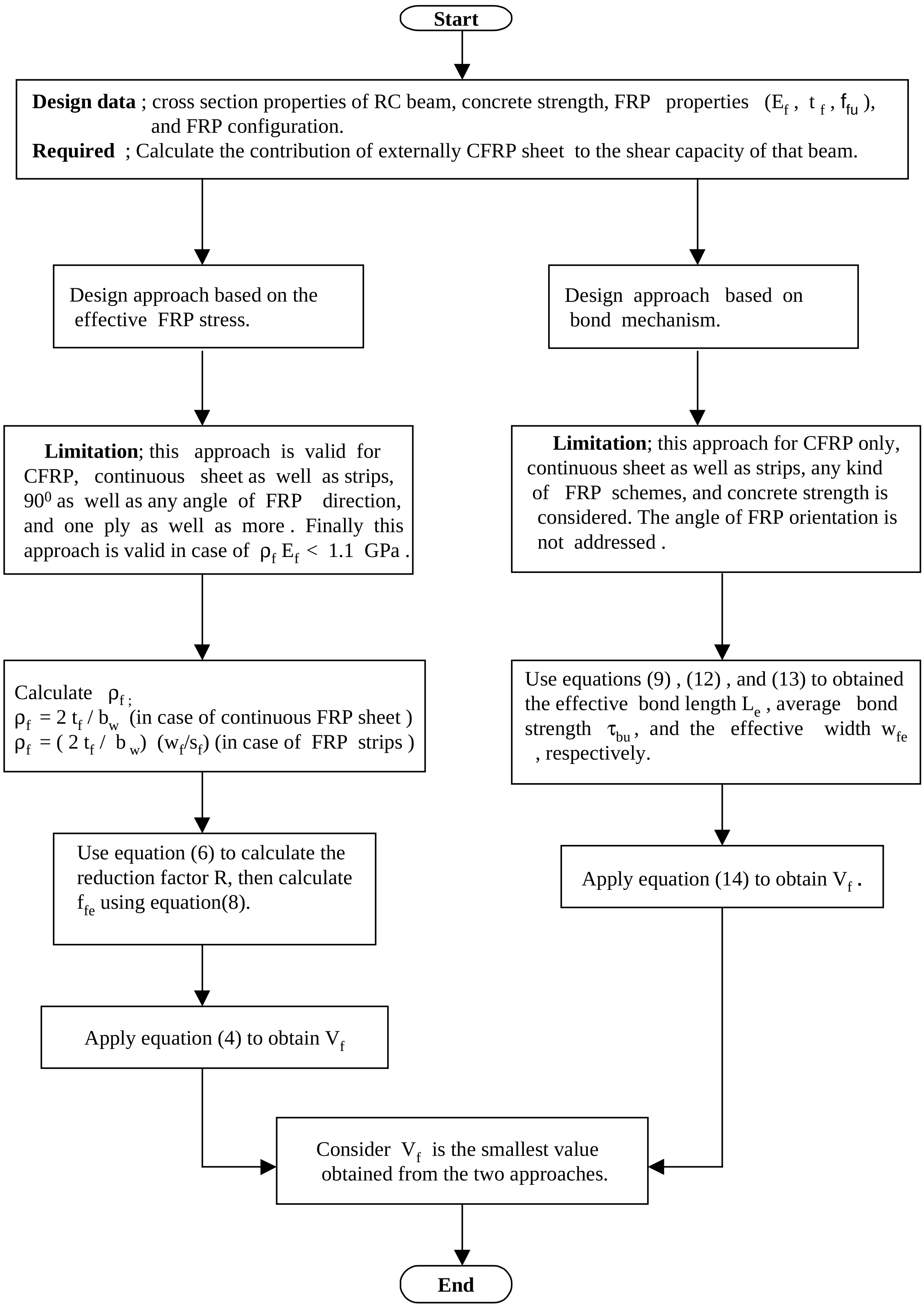

Fig . 9 . The Flowchart for the Calculation Sequence